

10.3.7 Integrovaní substituční metodou II

Předpoklady: 10306

Př. 1: Vypočti:

a) $\int \frac{3}{\sqrt{5x-1}} dx$

b) $\int x^2 \sqrt[3]{1-x^3} dx$

c) $\int \frac{1}{2-5x} dx$

a)

$$\int \frac{3}{\sqrt{5x-1}} dx = \int \frac{1}{5} \cdot \frac{3}{\sqrt{5x-1}} 5dx = \frac{3}{5} \int \frac{1}{\sqrt{t}} dt = \frac{3}{5} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{6}{5} \sqrt{t} + C = \frac{6}{5} \sqrt{5x-1} + C$$

$$t = 5x-1 \Rightarrow dt = 5 dx$$

b)

$$\int x^2 \sqrt[3]{1-x^3} dx = \int \left(-\frac{1}{3}\right) (-3x^2) \sqrt[3]{1-x^3} dx = -\frac{1}{3} \int \sqrt[3]{t} dt = -\frac{1}{3} \frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C = -\frac{1}{4} \sqrt[3]{t^4} + C$$

$$t = 1-x^3 \Rightarrow dt = -3x^2 dx$$

$$= -\frac{1}{4} \sqrt[3]{(1-x^3)^4} + C$$

c)

$$\int \frac{1}{2-5x} dx = \int \left(-\frac{1}{5}\right) \frac{1}{2-5x} (-5) dx = -\frac{1}{5} \int \frac{1}{t} dt = -\frac{1}{5} \ln|t| + C = -\frac{1}{5} \ln|2-5x| + C$$

$$= t = 2-5x \Rightarrow dt = -5 dx$$

Př. 2: Vypočti:

a) $\int \cos^3 x \cdot \sin x dx$

b) $\int \frac{\ln x}{2x} dx$

c) $\int \frac{3}{(3-2x)^5} dx$

$$\dots \text{ a) } \int \cos^3 x \cdot \sin x dx = -\int \cos^3 x \cdot (-\sin x) dx = -\int t^3 dt = -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$$

$$t = \cos x \Rightarrow dt = -\sin x dx$$

$$\text{ b) } \int \frac{\ln x}{2x} dx = \frac{1}{2} \int \ln x \frac{1}{x} dx = \frac{1}{2} \int t dt = \frac{1}{2} \frac{t^2}{2} + C = \frac{\ln^2 x}{4} + C$$

$$t = \ln x \Rightarrow dt = \frac{1}{x} dx$$

$$\text{ c) } \int \frac{3}{(3-2x)^5} dx = -\frac{3}{2} \int \frac{1}{(3-2x)^5} (-2) dx = -\frac{3}{2} \int \frac{1}{t^5} dt = -\frac{3}{2} \frac{t^{-4}}{-4} = \frac{3}{8(3-2x)^4}$$

$$t = 3-2x \Rightarrow dt = -2 dx$$

Př. 3: Vypočti:

a) $\int \sin(2x - \pi) dx$

b) $\int ae^{-x} dx$

c) $\int \cos\left(\frac{2}{3}x + \frac{\pi}{2}\right) dx$

a) $\int \sin(2x - \pi) dx = \frac{1}{2} \int \sin(2x - \pi) 2 dx = \frac{1}{2} \int \sin t dt = \frac{1}{2} (-\cos t) + C = -\frac{1}{2} \cos(2x - \pi) + C$

$t = 2x - \pi \Rightarrow dt = 2 dx$

b) $\int ae^{-x} dx = -a \int e^{-x} (-1) dx = -a \int e^t dt = -a \cdot e^t + C = -a \cdot e^{-x} + C$

$t = -x \Rightarrow dt = (-1) dx$

c) $\int \cos\left(\frac{2}{3}x + \frac{\pi}{2}\right) dx = \frac{3}{2} \int \cos\left(\frac{2}{3}x + \frac{\pi}{2}\right) \frac{2}{3} dx = \frac{3}{2} \int \cos t dt = \frac{3}{2} \sin t + C = \frac{3}{2} \sin\left(\frac{2}{3}x + \frac{\pi}{2}\right) + C$

$t = \frac{2}{3}x + \frac{\pi}{2} \Rightarrow dt = \frac{2}{3} dx$

Př. 4: Petáková:

strana 164/cvičení 89 b) d) f)

Shrnutí: Substituční metodu můžeme při integraci použít pokud se nám podaří vytvořit v integrálu derivaci substituované funkce.