2.2.9 Derivace funkcí (shrnutí)

Př. 1: Urči derivace:

a) $\left(x^3-2\sin x\right)'=3x^2-2\cos x$ ´

b) $\left(\frac{4}{x^3}\right)'=\left(4x^{-3}\right)'=4\cdot(-3)x^{-4}=-\frac{12}{x^4}$

c) $\left(2^x+\sqrt{x^3}\right)'=2^x\ln 2+\left(x^{\frac{3}{3}}\right)'=2^x\ln 2+\frac{3}{2}x^{\frac{1}{2}}=2^x\ln 2+\frac{3}{2}\sqrt{x}$

d) $\left(3\ln x-\operatorname{tg} x\right)'=3\cdot\frac{1}{x}-\frac{1}{\cos^2 x}=\frac{3}{x}-\frac{1}{\cos^2 x}$

Př. 2: Urči derivace:

a) $\left(x\cdot\log_3 x\right)'=x'\log_3 x+x\left(\log_3 x\right)'=\log_3 x+x\frac{1}{x\ln 3}=\log_3 x+\frac{1}{\ln 3}$

b) $\left(x\cdot\sqrt[3]{x^2}\right)'=\left(x^1\cdot x^{\frac{2}{3}}\right)'=\left(x^{\frac{5}{3}}\right)'=\frac{5}{3}x^{\frac{2}{3}}=\frac{5}{3}\sqrt[3]{x^2}$

c) $\left(\frac{x^2-4}{x+2}\right)'=\left(\frac{(x-2)(x+2)}{x+2}\right)'=(x-2)'=1$

d) $\left(\frac{x^2+4}{x+2}\right)'=\frac{\left(x^2+4\right)'(x+2)-\left(x^2+4\right)(x+2)'}{(x+2)^2}=\frac{2x(x+2)-\left(x^2+4\right)}{(x+2)^2}=$

$=\frac{2x^2+4x-x^2-4}{(x+2)^2}=\frac{x^2+4x-4}{(x+2)^2}$

Př. 3: Urči derivace:

a) $\left(x^3-2x^2+3\right)'=3x^2-2\cdot 2x+0=3x^2-4x$

b) $\left(\frac{1}{x^3-2x^2+3}\right)'=\left[\left(x^3-2x^2+3\right)^{-1}\right]'=-\frac{1}{\left(x^3-2x^2+3\right)^2}\left(x^3-2x^2+3\right)'=-\frac{3x^2-4x}{\left(x^3-2x^2+3\right)^2}$

c) $\left[\frac{\sqrt{x}\left(2x-\sqrt[3]{x^4}\right)}{\sqrt[3]{x}}\right]'=\left[\frac{2x\cdot x^{\frac{1}{2}}-x^{\frac{4}{3}}x^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right]'=\left[\frac{2x^{\frac{3}{2}}-x^{\frac{11}{6}}}{x^{\frac{1}{3}}}\right]'=\left[2x^{\frac{3}{2}-\frac{1}{3}}-x^{\frac{11}{6}-\frac{1}{3}}\right]=$

$=\left(2x^{\frac{7}{6}}-x^{\frac{3}{2}}\right)'=2\frac{7}{6}x^{\frac{1}{6}}-\frac{3}{2}x^{\frac{1}{2}}=\frac{7}{3}\sqrt[6]{x}-\frac{3}{2}\sqrt{x}$

Př. 4: Urči druhé derivace funkcí:

a) $y'=\left(x^3-2x^2+3\right)'=3x^2-4x$ $y''=\left(3x^2-4x\right)'=6x-4$

b) $y' = \left(x^2 \cdot e^x\right)' = \left(x^2\right)' e^x + x^2\left(e^x\right)' = 2xe^x + x^2e^x$

$y'' = \left(2xe^x + x^2e^x\right)' = (2x)' e^x + 2x\left(e^x\right)' + \left(x^2\right)' e^x + x^2\left(e^x\right)' = 2e^x + 4xe^x + x^2e^x$

c) $y = \log_2 x^2$

$y' = \left(\log_2 x^2\right)' = \frac{1}{x^2}\ln 2\left(x^2\right)' = \frac{\ln 2}{x^2}\cdot 2x = \frac{2\cdot\ln 2}{x} \qquad y'' = \left(\frac{2\cdot\ln 2}{x}\right)' = -\frac{2\cdot\ln 2}{x^2}$

Př. 5: Urči derivace:

a) $\left(x^2e^{\sin x}\right)' = \left(x^2\right)' e^{\sin x} + x^2\left(e^{\sin x}\right)' = 2x\cdot e^{\sin x} + x^2e^{\sin x}\cos x$

b) $\left(\frac{\sin x^2}{x^2+1}\right)' = \frac{\left(\sin x^2\right)'\left(x^2+1\right) - \sin x^2\left(x^2+1\right)'}{\left(x^2+1\right)^2} = \frac{\cos x^2\cdot 2x\left(x^2+1\right) - \sin x^2\cdot 2x}{\left(x^2+1\right)^2}$

c)

$$\left(\sqrt{x+\sqrt{x^2+2x}}\right)' = \frac{1}{2}\frac{1}{\sqrt{x+\sqrt{x^2+2x}}}\cdot\left(x+\sqrt{x^2+2x}\right)' =$$

$$= \frac{1}{2}\frac{1}{\sqrt{x+\sqrt{x^2+2x}}}\cdot\left(1+\frac{1}{2}\frac{1}{\sqrt{x^2+2x}}\left(x^2+2x\right)'\right) = \frac{1}{2}\frac{1}{\sqrt{x+\sqrt{x^2+2x}}}\cdot\left(1+\frac{2x+2}{2\sqrt{x^2+2x}}\right)$$

Př. 6: Urči derivace:

a) $\left(e^{\sin x\cdot x}\right)' = e^{\sin x\cdot x}\left(\sin x\cdot x\right)' = e^{\sin x\cdot x}\left[\left(\sin x\right)'\cdot x + \sin x\left(x\right)'\right] = e^{\sin x\cdot x}\left(\cos x\cdot x + \sin x\right)$

b)

$$\left[\sin\left(\frac{x^2+1}{2+x}\right)\right]' = \cos\left(\frac{x^2+1}{2+x}\right)\left(\frac{x^2+1}{2+x}\right)' = \cos\left(\frac{x^2+1}{2+x}\right)\left[\frac{\left(x^2+1\right)'(2+x) - \left(x^2+1\right)(2+x)'}{(2+x)^2}\right] =$$

$$= \cos\left(\frac{x^2+1}{2+x}\right)\frac{2x(2+x) - \left(x^2+1\right)}{(2+x)^2} = \cos\left(\frac{x^2+1}{2+x}\right)\frac{2x^2+4x-x^2-1}{(2+x)^2} = \cos\left(\frac{x^2+1}{2+x}\right)\frac{x^2+4x-1}{(2+x)^2}$$

c)

$$\left(\frac{1}{\sin^2(2x)+x^2-1}\right)' = -\frac{1}{\left[\sin^2(2x)+x^2-1\right]^2}\left[\sin^2(2x)+x^2-1\right]' =$$

$$= -\frac{1}{\left[\sin^2(2x)+x^2-1\right]^2}\left[2\sin(2x)\left(\sin[2x]\right)' + 2x\right] =$$

$$= -\frac{1}{\left[\sin^2(2x)+x^2-1\right]^2}\left[2\sin(2x)\cos(2x)(2x)' + 2x\right] =$$

$$= -\frac{1}{\left[\sin^2(2x)+x^2-1\right]^2}\left[2\sin(2x)\cos(2x)\cdot 2 + 2x\right]$$