2.9.26 Logaritmické rovnice (shrnuti)

Př. 1: Vyřeš rovnici $\log x^2 + \log x^4 = 3$.

Podmínky: $x \neq 0$. $\log x^2 + 2\log x^2 = 3$ $3\log x^2 = 3$ $\log x^2 = 1$

$\log x^2 = \log 10$ $x^2 = 10$ $x_1 = \sqrt{10}$ $x_2 = -\sqrt{10}$ $K = \{\pm\sqrt{10}\}$

Př. 2: Vyřeš rovnici $\frac{\log x - 2}{\log x + 1} = \frac{\log x + 3}{\log x - 1}$.

Podmínky: $x > 0$.

Substituce: $y = \log x$ $\frac{y-2}{y+1} = \frac{y+3}{y-1}$ $/(y+1)(y-1)$ $(y-2)(y-1) = (y+3)(y+1)$

$y^2 - 2y - y + 2 = y^2 + 3y + y + 3$ $-3y = 4y + 1$ $-1 = 7y$ $y = -\frac{1}{7}$

$y = \log x = -\frac{1}{7}$ $\log x = \log 10^{-\frac{1}{7}}$ $x = 10^{-\frac{1}{7}} = \frac{1}{\sqrt[7]{10}}$ $K = \left\{\frac{1}{\sqrt[7]{10}}\right\}$

Př. 3: Vyřeš rovnici $x^{\log x - 3} = \frac{x}{1000}$.

Podmínky: $x > 0$.

$\log x^{\log x - 3} = \log \frac{x}{1000}$ $(\log x - 3)\log x = \log x - \log 1000$ $(\log x - 3)\log x = \log x - 3$

Substituce: $y = \log x$ $(y-3)y = y - 3$ $y^2 - 3y = y - 3$ $y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$ $y_1 = 3$ $y_2 = 1$

$y = \log x = 3$ $\log x = \log 10^3$ $x = 1000$ $y = \log x = 1$ $\log x = \log 10^1$ $x = 10$

$K = \{10; 1000\}$

Př. 4: Vyřeš rovnici $2\log x^3 - 3\log\sqrt{x} + \log 2 = 2\log x + \log 4 - \frac{1}{2}\log x$.

Podmínky: $x > 0$.

$2\log x^3 - 3\log\sqrt{x} + \log 2 = 2\log x + \log 4 - \frac{1}{2}\log x$

$\log\left(x^3\right)^2 - \log\left(\sqrt{x}\right)^3 + \log 2 = \log x^2 + \log 4 - \log\sqrt{x}$

$\log x^6 - \log x^{\frac{3}{2}} + \log 2 = \log x^2 + \log 4 - \log x^{\frac{1}{2}}$ $\log \frac{2x^6}{x^{\frac{3}{2}}} = \log \frac{4x^2}{x^{\frac{1}{2}}}$

$\frac{2x^6}{x^{\frac{3}{2}}} = \frac{4x^2}{x^{\frac{1}{2}}}$ $\frac{x^4}{x\sqrt{x}} = \frac{2}{\sqrt{x}}$ $x^3 = 2$ $x = \sqrt[3]{2}$ $K = \{\sqrt[3]{2}\}$

Př. 5: Vyřeš rovnici: $\log_9^2 9x^2 + \log_9 81x^3 = 5$.

Podmínky: $x > 0$.

$\left(\log_9 9x^2\right)^2 + \log_9 81x^3 = 5 \qquad \left(\log_9 9 + \log_9 x^2\right)^2 + \log_9 81 + \log_9 x^3 = 5$

$(1 + 2\log_9 x)^2 + 2 + 3\log_9 x = 5$

Substituce: $y = \log_9 x \qquad (1+2y)^2 + 2 + 3y = 5 \qquad 4y^2 + 7y - 2 = 0$

$y_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 4 \cdot (-2)}}{2 \cdot 4} = \frac{-7 \pm 9}{8} \quad y_1 = \frac{-7-9}{8} = -2 \qquad y_2 = \frac{-7+9}{8} = \frac{1}{4}$

$y_1 = \log_9 x_1 = -2 \quad \log_9 x_1 = \log_9 9^{-2} \quad x_1 = \frac{1}{81} \qquad y = \log_9 x_2 = \frac{1}{4} \qquad x_2 = 3^{\frac{1}{2}} = \sqrt{3}$

$K = \left\{\frac{1}{81}; \sqrt{3}\right\}$

Př. 6: Vyřeš rovnici: $\ln(3x-1) \cdot \log_2\left(x^2\right) \cdot \log_{0,5}(3-x) = 0$.

Podmínky: $3x - 1 > 0 \Rightarrow x > \frac{1}{3}$, $x^2 > 0 \Rightarrow x \neq 0$, $3 - x > 0 \Rightarrow x < 3$.

$\ln(3x-1) = \ln e^0 \qquad \log_2\left(x^2\right) = \log_2 2^0 \qquad \log_{0,5}(3-x) = \log_{0,5} 0{,}5^0$

$3x - 1 = 1 \quad 3x = 2 \quad x_1 = \frac{2}{3} \qquad x^2 = 1 \quad x_2 = 1 \qquad x_3 = -1 \qquad 3 - x = 1 \qquad x_4 = 2$

Při kontrole podmínek jeden z kořenů $x_3 = -1$ vypadne: $K = \left\{\frac{2}{3}; 1; 2\right\}$

Př. 7: Vyřeš rovnici: $\ln\left\{\log_{0,5}\left[\log_\pi(\log_3 x - 1) + 2\right] + 2\right\} = 0$.

$\ln\left\{\log_{0,5}\left[\log_\pi(\log_3 x - 1) + 2\right] + 2\right\} = \ln 1 \quad \log_{0,5}\left[\log_\pi(\log_3 x - 1) + 2\right] + 2 = 1$

$\log_{0,5}\left[\log_\pi(\log_3 x - 1) + 2\right] = -1 = \log_{0,5} 0{,}5^{-1} \quad \log_{0,5}\left[\log_\pi(\log_3 x - 1) + 2\right] = \log_{0,5} 2$

$\log_\pi(\log_3 x - 1) + 2 = 2 \qquad \log_\pi(\log_3 x - 1) = 0 = \log_\pi \pi^0 \qquad \log_\pi(\log_3 x - 1) = \log_\pi 1$

$\log_3 x - 1 = 1 \quad \log_3 x = 2 = \log_3 3^2 \quad \log_3 x = \log_3 9 \qquad x = 9 \qquad K = \{9\}$

Př. 8: Vyřeš rovnici: $\log_x 3 + 3\log_{3x} 9 = 6\log_{x^2} 3$.

Podmínky: $x > 0$, $x \neq 1$.

$\log_x 3 + 3\log_{3x} 9 = 6\log_{x^2} 3 \qquad \frac{\log_3 3}{\log_3 x} + 3\frac{\log_3 9}{\log_3 3x} = 6\frac{\log_3 3}{\log_3 x^2}$

$\frac{1}{\log_3 x} + 3\frac{2}{\log_3 3 + \log_3 x} = 6\frac{1}{2\log_3 x} \qquad \frac{1}{\log_3 x} + \frac{6}{1 + \log_3 x} = \frac{3}{\log_3 x}$

Substituce: $y = \log_3 x \qquad \frac{1}{y} + \frac{6}{1+y} = \frac{3}{y} \quad / \, y(1+y) \qquad 1 + y + 6y = 3(1+y)$

$1 + y + 6y = 3 + 3y \qquad 4y = 2 \qquad y = \frac{1}{2}$

$y = \log_3 x = \frac{1}{2} \qquad \log_3 x = \log_3 3^{\frac{1}{2}} \qquad x = \sqrt{3} \qquad K = \left\{\sqrt{3}\right\}$