

2.9.6 Exponenciální rovnice III

Př. 1: Vyřeš rovnici $2^{x-1} \cdot \left(\frac{1}{3}\right)^{x-1} - 2^x \cdot \left(\frac{1}{3}\right)^x = \frac{2}{9}$.

$$\left(\frac{2}{3}\right)^{x-1} - \left(\frac{2}{3}\right)^x = \frac{2}{9} \quad \left(\frac{2}{3}\right)^{-1} \cdot \left(\frac{2}{3}\right)^x - \left(\frac{2}{3}\right)^x = \frac{2}{9}$$

Substitute: $\left(\frac{2}{3}\right)^x = a \Rightarrow \left(\frac{3}{2}\right)a - a = \frac{2}{9} \quad \frac{1}{2}a = \frac{2}{9} \quad a = \frac{4}{9}$

$$a = \left(\frac{2}{3}\right)^x = \frac{4}{9} \quad \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^2 \quad x = 2 \quad K = \{2\}$$

Př. 2: Vyřeš rovnici $20 \cdot 2^x - 2^{x+1} = 3^{x+2} - 3^x$.

$$20 \cdot 2^x - 2^{x+1} = 3^{x+2} - 3^x \quad /: 3^x \quad 20 \cdot \frac{2^x}{3^x} - \frac{2^{x+1}}{3^x} = \frac{3^{x+2}}{3^x} - \frac{3^x}{3^x} \quad 20 \cdot \left(\frac{2}{3}\right)^x - 2 \left(\frac{2}{3}\right)^x = 9 \cdot 1 - 1$$

Substitute: $\left(\frac{2}{3}\right)^x = a \Rightarrow 20a - 2a = 8 \cdot 1 \quad 18a = 8 \quad a = \frac{8}{18} = \frac{4}{9}$

$$a = \left(\frac{2}{3}\right)^x = \frac{4}{9} \quad \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^2 \quad x = 2 \quad K = \{2\}$$

Př. 3: Vyřeš rovnici $3 \cdot 5^{x-2} + 7 \cdot 5^{x-3} = 5 \cdot 3^{x-3} + 5^{x-1}$.

$$3 \cdot 5^{x-2} + 7 \cdot 5^{x-3} = 5 \cdot 3^{x-3} + 5^{x-1} \quad /: 5^{x-3} \quad 3 \cdot \frac{5^{x-2}}{5^{x-3}} + 7 \cdot \frac{5^{x-3}}{5^{x-3}} = 5 \cdot \frac{3^{x-3}}{5^{x-3}} + \frac{5^{x-1}}{5^{x-3}}$$

$$3 \cdot 5 + 7 \cdot 1 = 5 \cdot \left(\frac{3}{5}\right)^{x-3} + 25 \quad -3 = 5 \cdot \left(\frac{3}{5}\right)^{x-3} \quad -\frac{3}{5} = \left(\frac{3}{5}\right)^{x-3} \quad K = \emptyset$$

Př. 4: Vyřeš rovnici $3 \cdot 2^{2-x} + 2 \cdot 2^{1-x} = 8 \cdot 3^{2-x} + 3 \cdot 3^{1-x}$.

$$3 \cdot 2^{2-x} + 2 \cdot 2^{1-x} = 8 \cdot 3^{2-x} + 3 \cdot 3^{1-x} \quad /: 3^{2-x} \quad 3 \cdot \frac{2^{2-x}}{3^{2-x}} + 2 \cdot \frac{2^{1-x}}{3^{2-x}} = 8 \cdot \frac{3^{2-x}}{3^{2-x}} + 3 \cdot \frac{3^{1-x}}{3^{2-x}}$$

$$3 \cdot \left(\frac{2}{3}\right)^{2-x} + 2 \cdot \frac{2^{2-x-1}}{3^{2-x}} = 8 \cdot 1 + 3 \cdot \frac{1}{3} \quad 3 \cdot \left(\frac{2}{3}\right)^{2-x} + 2 \cdot \frac{1}{2} \left(\frac{2}{3}\right)^{2-x} = 8 \cdot 1 + 3 \cdot \frac{1}{3}$$

Substitute: $\left(\frac{2}{3}\right)^{2-x} = a \Rightarrow 3a + a = 9 \quad 4a = 9 \quad a = \frac{9}{4}$

$$a = \left(\frac{2}{3}\right)^{2-x} = \frac{9}{4} \quad \left(\frac{2}{3}\right)^{2-x} = \left(\frac{3}{2}\right)^2 = \left(\frac{2}{3}\right)^{-2} \quad 2-x = -2 \quad x = 4 \quad K = \{4\}$$

Př. 5: Vyřeš rovnici $2 \cdot 4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} + 2^{2x-1}$.

$$2 \cdot 2 \cdot 4^{x-\frac{1}{2}} - 3^{x-\frac{1}{2}} = 3 \cdot 3^{x-\frac{1}{2}} + 4^{x-\frac{1}{2}} \quad /: 4^{x-\frac{1}{2}} \quad 4 - \left(\frac{3}{4}\right)^{x-\frac{1}{2}} = 3 \cdot \left(\frac{3}{4}\right)^{x-\frac{1}{2}} + 1$$

Substitute: $\left(\frac{3}{4}\right)^{x-\frac{1}{2}} = a \Rightarrow 4 - a = 3a + 1 \quad 3 = 4a \quad a = \frac{3}{4}$

$$a = \left(\frac{3}{4}\right)^{x-\frac{1}{2}} = \frac{3}{4} \quad x - \frac{1}{2} = 1 \quad x = \frac{3}{2} \quad K = \left\{\frac{3}{2}\right\}$$

Př. 6: Vyřeš rovnici $9^{x+1} + 5 \cdot 6^x = 4^{x+1}$.

$$9 \cdot 9^x + 5 \cdot 2^x \cdot 3^x = 4 \cdot 4^x \quad 9 \cdot 3^x \cdot 3^x + 5 \cdot 2^x \cdot 3^x = 4 \cdot 2^x \cdot 2^x$$

$$9 \cdot 3^x \cdot 3^x + 5 \cdot 2^x \cdot 3^x = 4 \cdot 2^x \cdot 2^x \quad /: (2^x \cdot 2^x) \quad 9 \cdot \left(\frac{3}{2}\right)^x \cdot \left(\frac{3}{2}\right)^x + 5 \cdot \left(\frac{3}{2}\right)^x = 4 \cdot 1$$

Substitute: $\left(\frac{3}{2}\right)^x = a \Rightarrow 9a \cdot a + 5a = 4 \cdot 1 \quad 9a^2 + 5a - 4 = 0$

$$a_{1,2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 9 \cdot (-4)}}{2 \cdot 9} = \frac{-5 \pm 13}{18} \quad a_1 = \frac{-5 + 13}{18} = \frac{8}{18} = \frac{4}{9} \quad a_2 = \frac{-5 - 13}{18} = -1$$

$$a_1 = \left(\frac{3}{2}\right)^{x_1} = \frac{4}{9} \quad \left(\frac{3}{2}\right)^{x_1} = \left(\frac{3}{2}\right)^{-2} \quad a_2 = \left(\frac{3}{2}\right)^{x_2} = -1$$

$$x_1 = -2$$

$$K = \{-2\}$$

Př. 7: Vyřeš rovnici $4 \cdot 16^x + 4^{x+1} = 2 \cdot 8^{x+1} + 2^{3x}$.

$$4 \cdot 16^x + 4^{x+1} = 2 \cdot 8^{x+1} + 2^{3x} \quad 4 \cdot (2^4)^x + (2^2)^{x+1} = 2 \cdot (2^3)^{x+1} + 2^{3x}$$

$$4 \cdot 2^{4x} + 2^{2x+2} = 2 \cdot 2^{3x+3} + 2^{3x} \quad 4 \cdot 2^{4x} + 2^2 \cdot 2^{2x} = 2 \cdot 2^3 \cdot 2^{3x} + 2^{3x}$$

$$4 \cdot 2^{4x} + 4 \cdot 2^{2x} = 16 \cdot 2^{3x} + 2^{3x} \quad /: 2^{2x} \quad 4 \cdot 2^{2x} + 4 = 16 \cdot 2^x + 2^x$$

Substitute: $2^x = a \Rightarrow 4a^2 + 4 = 16a + a \quad 4a^2 - 17a + 4 = 0$

$$a_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-17) \pm \sqrt{(-17)^2 - 4 \cdot 4 \cdot 4}}{2 \cdot 4} = \frac{17 \pm \sqrt{225}}{8} = \frac{17 \pm 15}{8}$$

$$a_1 = \frac{17+15}{8} = \frac{32}{8} = 4 \quad a_2 = \frac{17-15}{8} = \frac{1}{4}$$

Návrat k původní proměnné:

$$a_1 = 2^{x_1} = 4 \quad 2^{x_1} = 2^2 \quad x_1 = 2 \quad a_2 = 2^{x_2} = \frac{1}{4} \quad 2^{x_2} = 2^{-2} \quad x_2 = -2$$

$$K = \{-2; 2\}$$

Př. 8: Petáková:

strana 34/cvičení 4 b) c) d)

strana 34/cvičení 5 a) b)